International Journal of Engineering Materials and Manufacture (2018) 3(1) 9-17
https://doi.org/10.26776/ijemm.03.01.2018.02

Stress Analysis of Thin-Walled Laminated Composite Beams under Shear and Torsion

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Received: 09 December 2017
Accepted: 29 December 2017
Published: 30 March 2018
Publisher: Deer Hill Publications

ABSTRACT

An educational software which can aid students in the stress analysis of thin wall open sections made of composite material has been developed. The software enables students to easily calculate stresses in different shapes of thin walled open section and evaluate the stresses in each ply under shear and torsion. Results obtained through this software have been validated against ANSYS. The software is intended to be an educational tool for effective teaching and learning process on thin-walled structures, aircraft structures and composite structures courses.

Keywords: Educational software; Stress analysis; Thin wall open section; Composite structures

1 INTRODUCTION

Composite materials are well known to have excellent fatigue resistance, high specific strength and stiffness, good corrosion resistance, excellent fire resistance and lower thermal expansion. In early years of development of composite materials in aerospace application, they were only used for secondary and tertiary structure of the aircraft. However, recently, the airframe and primary structures are also made entirely of composite. Moreover in aerospace industry, most of the aircraft structures are thin-walled in nature and open section beams are used in aircrafts to stiffen the thin skins of the cellular components and provide support for internal loads from floors, engine mountings, etc. Thus the focus of this study is on the stress analysis of composite thin-walled open sections.

Literature on stress analysis of thin walled sections under different type of loadings are numerous. Parambil (2010) presented a closed form solution for ply by ply stress analysis of thin-walled beam with un-symmetrical cross-section I- beam and validated with respect to ANSYS. Nurhuda and Ali (2017) developed an educational software for thin-walled sections of isotropic and composite materials. The software evaluates only the average normal stresses due to bending and average shear stresses due to shear and torsion for I, C, T, and Z cross-section beams. Thus layerwise stress analysis was not carried out in this study.

Vishal (2012) extended the research done by Parambil (2010) in stress analysis of laminated composite beam under torsional loads for I-beam. He has done the analysis to calculate shear center, equivalent torsional stiffness, equivalent warping stiffness and equivalent bending stiffness and compared with ANSYS.

Kollar and Pluszik (2012) introduced a new theory called torsional-warping shear deformation theory in order to determine the stiffness of anisotropic beams. The theory also includes the restrained torsional warping from Vlasov's theory and the in-plane shear deformations from Timoshenko theory. Hubert (2013) studied on thin-walled composite column under axial compression. Lachenal et al. (2013) introduced a new feature to I-composite beam by re-designing and adding a morphing twist functionality to the beam. The flanges layup were designed to obtain a highly non-linear torsional stiffness whilst the web are designed for lower stiffness in order to allow large twist deformation.

Carpinteri et al. (2014) studied the effect of warping of U-shaped beam under torsion and shear force. Li and Easterbrook (2014) developed a simple way to derive the torsional equations for thin-walled open or closed or combined open and closed sections beam under free torsion using a simple statically indeterminate concept. Sachidananda (2016) in his work presented stress analysis of unsymmetrical layup of composite I-beam. Ply stresses due to axial loads were presented and compared to FEA analysis.

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Reference: Ali, J. S. M., Meftah, H. and Masturah, M. (2018). Stress Analysis of Thin-Walled Laminated Composite Beams. *International Journal of Engineering Materials and Manufacture*, 3(1), 9-17

Through the literature survey, it can be concluded that many scholars have derived the closed form solutions for thin-walled laminated structures, where most of them chose the laminated plate, I- beam and box beam as subject of their researches. Very few have carried out the ply stress analysis and have tabulated results which are useful for validation. Furthermore, it was found that software based on closed form solution is not available for ply stress analysis of thin walled sections, which motivated to develop such an educational software.

The present work is the continuation of work done by Ali et al [2015] which focused on I, C and T- open section of thin-walled laminated composite beams under axial loads and bending moments only. In this work it has been extended for the shear and torsional loading. The software has been developed in MATLAB which is able to calculate ply stresses (σ_x, σ_y, τ_{xy}), and the results were validated against FEA software (ANSYS). In this paper, only I, C and T-beams which are subjected to torsional loads and shear force is presented. The software is valid for beams which are long, un-tapered and slender with ratio of the cross-sectional dimension to the length is at least 1/10. Meanwhile, the ratio of the laminate thickness to the cross-sectional dimension is at least 1/10. Present software utilizes a beam theory developed by Kollar and Pluszik (2002) which neglects the effect of shear deformation and restrained warping.

2 METHODOLOGY

A cantilever beam made up of thin-walled laminated open section made of composite material subjected to shear force and torsional moment is considered for analysis. The following laminate theory with general laminate constitutive equations and the theory of laminated beam analysis are from Kollar and Pluzsik (2002) and Datoo (1991). The lay-up of each wall segment is symmetrical, however every wall segment can have different lay-up configuration.

2.1 Determination of Elastic Constants of Laminate

The procedure for computing the stiffness, compliance and equivalent elastic constants, which is similar for any laminate configuration, is as follows:

Firstly, determine the reduced stiffness terms Q_{ij} and determine the transformed reduced stiffness terms $\bar{Q}_{ij}$ using the Q_{ij} terms.

The stress-strain relationship in the reduced stiffness terms is shown in Eq. (1). Meanwhile, the total of direct and shear strain is shown in Eq. (2).

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix}=\begin{bmatrix}\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{13}\\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{23}\\ \bar{Q}_{13} & \bar{Q}_{23} & \bar{Q}_{33}\end{bmatrix}\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{bmatrix} \tag{1}$$

$$\begin{bmatrix}\epsilon_x\\ \epsilon_y\\ \gamma_{xy}\end{bmatrix}=\begin{bmatrix}\epsilon_x^0\\ \epsilon_y^0\\ \gamma_{xy}^0\end{bmatrix}+zk\begin{bmatrix}k_x\\ k_y\\ k_{xy}\end{bmatrix} \tag{2}$$

Determine the laminate stiffness terms A_{ij}, B_{ij} and D_{ij}. Force and moment intensities can be determined as shown in Eq. (3).

$$\begin{bmatrix}N_x\\ N_y\\ N_{xy}\\ M_x\\ M_y\\ M_{xy}\end{bmatrix}=\begin{bmatrix}A_{11} & A_{12} & A_{13} & \vdots & B_{11} & B_{12} & B_{13}\\ A_{12} & A_{22} & A_{23} & & B_{12} & B_{22} & B_{23}\\ A_{13} & A_{23} & A_{33} & & B_{13} & B_{23} & B_{33}\\ B_{11} & B_{12} & B_{13} & & D_{11} & D_{12} & D_{13}\\ B_{12} & B_{22} & B_{23} & & D_{12} & D_{22} & D_{23}\\ B_{13} & B_{23} & B_{33} & & D_{13} & D_{23} & D_{33}\end{bmatrix}\begin{bmatrix}\epsilon_x^0\\ \epsilon_y^0\\ \gamma_{xy}^0\\ k_x\\ k_y\\ k_{xy}\end{bmatrix} \tag{3}$$

The laminate stiffness terms A_{ij}, B_{ij} and D_{ij} are inverted to obtain the corresponding compliance terms a_{ij}, b_{ij} and d_{ij}.

$$\begin{bmatrix}\epsilon_x^0\\ \epsilon_y^0\\ \gamma_{xy}^0\\ k_x\\ k_y\\ k_{xy}\end{bmatrix}=\begin{bmatrix}a_{11} & a_{12} & a_{13} & \vdots & b_{11} & b_{12} & b_{13}\\ a_{12} & a_{22} & a_{23} & & b_{12} & b_{22} & b_{23}\\ a_{13} & a_{23} & a_{33} & & b_{13} & b_{23} & b_{33}\\ b_{11} & b_{12} & b_{13} & & d_{11} & d_{12} & d_{13}\\ b_{12} & b_{22} & b_{23} & & d_{12} & d_{22} & d_{23}\\ b_{13} & b_{23} & b_{33} & & d_{13} & d_{23} & d_{33}\end{bmatrix}\begin{bmatrix}N_x\\ N_y\\ N_{xy}\\ M_x\\ M_y\\ M_{xy}\end{bmatrix} \tag{4}$$

2.2 Analysis of Thin-walled Composite Open Sections

A thin-walled composite open section is made from an assembly of flat layered laminated composite. A section is defined as thin-walled if its thickness is small compared to the cross-sectional dimension; the ratio of the thickness to the cross-sectional dimension is at least one tenth. In this analysis, a cantilevered beam is considered which is fixed at one end subjected to either shear force or torsional load. The length of the beam is in x-direction, the height of the beam's cross-section is in z-direction and y-direction is the width of the beam's cross-section. Three different cross-sections are considered in this analysis which are I, C and T- sections as shown in figure 1. All laminates considered are symmetric and details of the layup of the laminates are presented in Tables 1 and 2.

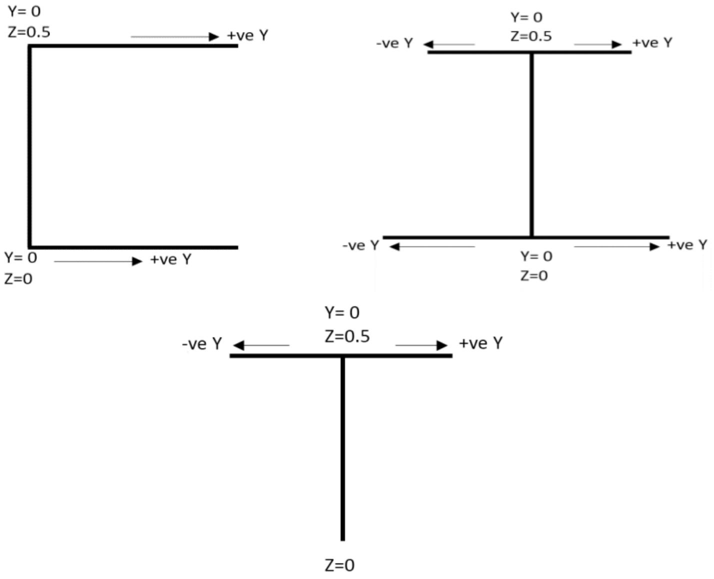


Figure 1: Cross-sections C, I and T of beam

2.2.1 Torsion

According to Kollar and Springer (2003), for an open-section beam under torsion, the in-plane forces per unit length which are N_y, N_{xy} and moment per unit length M_y are zeros. Thus, the expressions for N_x, M_x and M_{xy} in matrix form is:

$$\begin{bmatrix} N_x \\ M_x \\ M_{xy} \end{bmatrix} = [R\eng][\mu_k]^{-1}[R_k][W]\begin{bmatrix} N \\ m_y \\ m_z \\ T \end{bmatrix} \tag{5}$$

Where N, m_y, m_z and T are the applied force, moment and torque at the centroid. Under pure torsion, N, m_y and m_z are zero. The expressions of $R\eng$,μ_k, R_k and W are as follows:

$$[\boldsymbol{W}] = [\boldsymbol{P}]^{-1} \tag{6}$$

$$[\boldsymbol{P}] = \sum_{k=1}^{K}[\boldsymbol{R}_k]\,[\boldsymbol{R}_k]^T[\boldsymbol{\omega}_k]^{-1} \tag{7}$$

$$[\boldsymbol{R}_k] = \begin{bmatrix} 1 & z_k & y_k & 0 \\ 0 & \cos\alpha_k & -\sin\alpha_k & 0 \\ 0 & \sin\alpha_k & \cos\alpha_k & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8}$$

$$[\boldsymbol{\omega}_k] = \frac{1}{b_k}\begin{bmatrix} a_{11} & 0 & 0 & 0 \\ 0 & d_{11} & 0 & \frac{-d_{13}}{2} \\ 0 & 0 & \frac{12}{A_{11}{b^2}_k} & 0 \\ 0 & \frac{-d_{13}}{2} & 0 & \frac{d_{33}}{4} \end{bmatrix} \tag{9}$$

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{12} & A_{22} & A_{23} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & d_{11} & d_{13} \\ 0 & d_{13} & d_{33} \end{bmatrix}^{-1} \tag{10}$$

$$[\boldsymbol{R}\eng] = \begin{bmatrix} 1 & 0 & \eng & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix} \tag{11}$$

Where ŋ is the position along the wall segment.

$$[\boldsymbol{\mu_k}] = \begin{bmatrix} \boldsymbol{a_{11}} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{d_{11}} & \boldsymbol{d_{13}} \\ \boldsymbol{0} & \boldsymbol{d_{13}} & \boldsymbol{d_{33}} \end{bmatrix} \tag{12}$$

After obtaining the expressions for N_x, M_x and M_{xy}, these values can be substituted into equations (4) and (1). We are interested to find the value of maximum shear stress. Thus, the equation is reduced to:

$$\tau_{xy} = [\bar{Q}_{13} \quad \bar{Q}_{23} \quad \bar{Q}_{33}] \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix} \tag{13}$$

2.2.2 Shear Force

Expression for shear flow for open section under application of shear forces V_z and V_y (Kollar and Springer 2003):

$$\boldsymbol{q^{open} = -\frac{EI_{zz}V'_z - EI_{yz}V'_y}{EI_{yy}EI_{zz} - (EI_{yz})^2} \int_0^{s1} \frac{z}{a_{11}} ds - \frac{-EI_{yz}V'_z - EI_{yy}V'_y}{EI_{yy}EI_{zz} - (EI_{yz})^2} \int_0^{s1} \frac{y}{a_{11}} ds} \tag{14}$$

Average shear stress can be obtained by dividing shear flow by the wall thickness.

$$\tau_{xy} = \frac{q^{open}}{t} \tag{15}$$

2.3 ANSYS Simulation

The I, C, and T cross-section beams are modelled using ANSYS by using Shell 181 layered element. Meshing is done by setting the size of width lines to 0.01, meanwhile the length lines are set to 500 number of division. The spacing ratio is set to a negative value so that the density of the element will be increased at the edges. Prior to application of load, the model is bonded to each other using the Contact Wizard. One edge of the beam is constrained by setting all degree of freedom to zero. A typical ANSYS output window is shown in Figure 2 for the case of torsion of T-beam.

3 RESULTS AND DISCUSSIONS

The programming for the stress analysis components discussed in the previous section was carried out in MATLAB. Three different standard thin-walled cross-sections – C, I, and T-sections are considered in the analysis. User will be asked to enter material properties, cross-sectional of the beam, number of layers and their orientation. The software will compute the laminate stiffness properties based on the user input data as shown in Figure 3. For a given input of torsional moment, the output is the maximum shear stress in each ply. For stress analysis for shear force, user is required to enter any value of shear force and the software will generate the average shear stress. The length of the beam is taken to be 10 inches and all the tabulated results (Tables 3-9) are presented at the mid of the beam i.e. at x= 5 in. The y and z coordinates referred in the tables are as shown in Figure 1, with z=0 is at the mid plane of the laminate of flange for C and I section and it is at the bottom for T section.

3.1 Validation of the Results

The individual components of the software were checked to ensure its validity and accuracy by comparing the results with ANSYS. The material used in this analysis is T300/977-2 carbon/epoxy laminate. The elastic properties of the plies for all cross-section are: E_1 = 21.75(10^6) psi, E_2 = E_3=1.595(10^6) psi, G_{12} = G_{13} =0.8702(10^6) psi, G_{23} =0.5366(10^6) psi, ν_{12}= ν_{13}= ν_{23}= 0.25. All plies are 0.005 inch thick.

Table 1 and 2 below give the data of the geometrical cross-section (width for flange and height for web) and ply orientation of each sections respectively. Similar to Parambil (2015), for the top flange, the ply orientation is $[45/-45/0/90]_s$. for the lower flange, the ply orientation is $[45/-45/0_2/90]_s$ while for the web is [45/-45].

Table 1: Width of Geometrical Cross-section

Case	Upper flange (in)	Lower flange (in)	Web (in) (height)
I-Beam	0.5	0.75	0.5
C-Beam	0.5	0.5	0.5
T-Beam	1	N/A	0.5

Table 2: Ply Orientation

Case	Upper flange	Lower flange	Web
I-Beam	$(\pm45/0/90)_s$	$(\pm45/0_2/90)_s$	$(\pm45)_s$
C-Beam	$(0/\pm45/90)_s$	$(0/\pm45/90)_s$	$(\pm45)_s$
T-Beam	$(0/\pm45/90)_s$	N/A	$(\pm45)_s$

3.1.1 Beam Under Torsion

I, C and T- beams which are subjected to torsional moment value of 0.5 lb.in is analysed in this section. If a torsional moment is applied about x-axis, the beam will twist with a constant rate of twist. In ANSYS, one end is fixed and a constant rate of twist is applied at the other end. Typical results for shear stresses in each ply for flange and web of T section is given in Tables 3-4, whereas the Table 5-6 gives results for shear stresses in each ply for flange and web of C section. It can be found from the tables that the results from the present software agree well with the ANSYS results except for some cases. The error involved is attributed to the modelling error in ANSYS such as in the case of applying torsion, a statically equivalent load was applied in FEA.

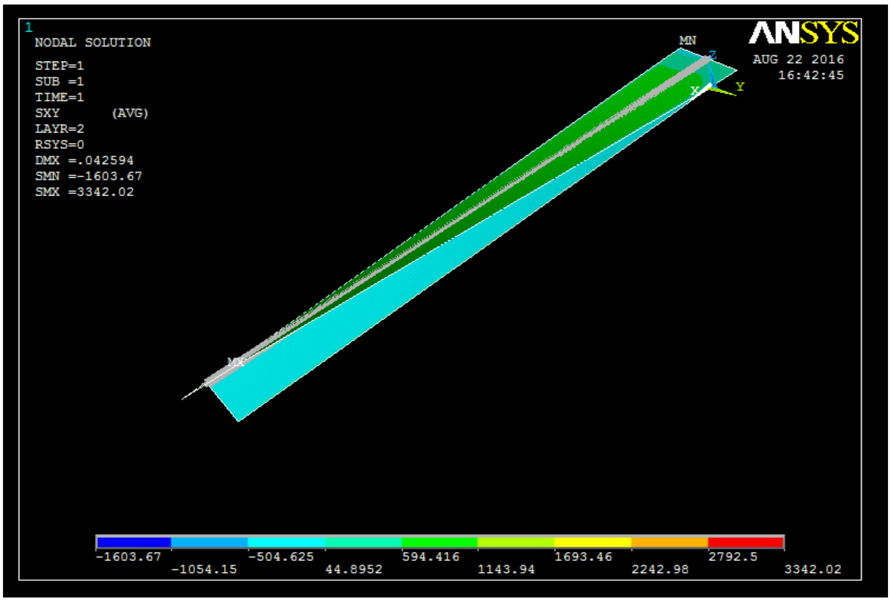
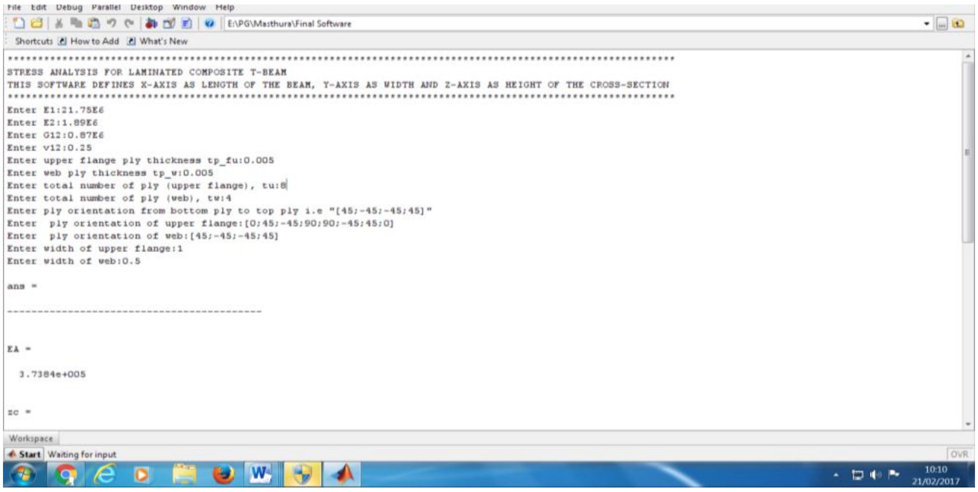


Figure 2: Typical ANSYS output window (shear stress SXY for T-beam on layer 2 under torsion)



```
****************************************************************************************************
STRESS ANALYSIS FOR LAMINATED COMPOSITE T-BEAM
THIS SOFTWARE DEFINES X-AXIS AS LENGTH OF THE BEAM, Y-AXIS AS WIDTH AND Z-AXIS AS HEIGHT OF THE CROSS-SECTION
****************************************************************************************************
Enter E1:21.75E6
Enter E2:1.89E6
Enter G12:0.87E6
Enter v12:0.25
Enter upper flange ply thickness tp_fu:0.005
Enter web ply thickness tp_w:0.005
Enter total number of ply (upper flange), tu:8
Enter total number of ply (web), tw:4
Enter ply orientation from bottom ply to top ply i.e "[45;-45;-45;45]"
Enter  ply orientation of upper flange:[0;45;-45;90;90;-45;45;0]
Enter  ply orientation of web:[45;-45;-45;45]
Enter width of upper flange:1
Enter width of web:0.5

ans =

------------------------------------------


EA =

  3.7384e+005


zc =
```

Figure 3: Typical input/output window for the MATLAB software

Table 3: Shear stress on the upper flange of T-beam due to torsion

X	Y	Z
5	0	0.5
PLY	Unit : lb/in2	SXY
1 (0)	ANSYS	227.63
	PRESENT SOFTWARE	216.947
	%DIFF	4.92
2 (45)	ANSYS	783.14
	PRESENT SOFTWARE	746.443
	%DIFF	4.92
3 (-45)	ANSYS	595.41
	PRESENT SOFTWARE	567.438
	%DIFF	4.93
4 (90)	ANSYS	0.00
	PRESENT SOFTWARE	0.00
	%DIFF	0.00
5 (90)	ANSYS	-75.875
	PRESENT SOFTWARE	-72.3158
	%DIFF	4.92
6 (-45)	ANSYS	-1194
	PRESENT SOFTWARE	-1138.06
	%DIFF	4.92
7 (45)	ANSYS	-1172.1
	PRESENT SOFTWARE	-1117.01
	%DIFF	4.93
8 (0)	ANSYS	-303.5
	PRESENT SOFTWARE	-289.263
	%DIFF	4.92

Table 4: Shear stress on the web of T-beam due to torsion

X	Y	Z
5	0	0.25
PLY	Unit : lb/in2	SXZ
1 (45)	ANSYS	241.58
	PRESENT SOFTWARE	236.016
	%DIFF	2.36
2 (-45)	ANSYS	4.2521
	PRESENT SOFTWARE	4.24759
	%DIFF	0.11
3 (-45)	ANSYS	-712.91
	PRESENT SOFTWARE	-696.679
	%DIFF	2.33
4 (45)	ANSYS	-495.91
	PRESENT SOFTWARE	-484.775
	%DIFF	2.30

Table 5: Shear stress on the lower flange of C-beam due to torsion

X	Y	Z
5	0.25	0
PLY	Unit : lb/in2	SXY
1 (0)	ANSYS	232.88
	PRESENT SOFTWARE	225.893
	%DIFF	3.09
2 (45)	ANSYS	832.35
	PRESENT SOFTWARE	774.849
	%DIFF	7.42
3 (-45)	ANSYS	614.99
	PRESENT SOFTWARE	593.354
	%DIFF	3.65
4 (90)	ANSYS	0.00
	PRESENT SOFTWARE	0.00
	%DIFF	0.00
5 (90)	ANSYS	-71.163
	PRESENT SOFTWARE	-75.2977
	%DIFF	5.49
6 (-45)	ANSYS	-1168
	PRESENT SOFTWARE	-1182.04
	%DIFF	1.19
7 (45)	ANSYS	-1142.4
	PRESENT SOFTWARE	-1166.17
	%DIFF	2.04
8 (0)	ANSYS	-299.2
	PRESENT SOFTWARE	-301.191
	%DIFF	0.66

Table 6: Shear stress on the web of C-beam due to torsion

X	Y	Z
5	0	0.25
PLY	Unit : lb/in2	SXZ
1 (45)	ANSYS	270.25
	PRESENT SOFTWARE	250.828
	%DIFF	7.74
2 (-45)	ANSYS	0.00
	PRESENT SOFTWARE	0.00
	%DIFF	0.00
3 (-45)	ANSYS	-781.81
	PRESENT SOFTWARE	-730.488
	%DIFF	7.03
4 (45)	ANSYS	-535.96
	PRESENT SOFTWARE	-499.682
	%DIFF	7.26

3.1.2 Beam Under Shear Force

The following section shows the results for I- section beam under -100lb shear force at the centre of the web. Typical results for average shear stresses for upper and lower flange (y=0) as well as on the web (z=0.25) is given in Tables 7-9. It can be found from the table that the results from the software agree well with the ANSYS with a reasonable modelling error. As the software for the case of shear loading produces the average shear stress as the output, the layerwise results of shear stresses from ANSYS were averaged to compare with the software thus leading to further error in shear loading case.

Table 7: Average shear stress on the lower flange of I-beam due to shear force

PLY	Unit : lb/in2	SXY
1(45)	ANSYS	-5289
2(-45)		12494
3(0)		556.31
4(0)		552.32
5(90)		548.32
6(90)		544.33
7(0)		540.34
8(0)		536.35
9(-45)		11518
10(45)		-4501.7
	ANSYS (AVERAGE)	1749.927
	PRESENT SOFTWARE	2003.85
	%DIFF	12.67

Table 8: Average shear stress on the upper flange of I-beam due to shear force

PLY	Unit : lb/in2	SXY
1(45)	ANSYS	15865
2(-45)		-23317
3(0)		-571.37
4(90)		-577.71
5(90)		-584.05
6(0)		-590.39
7(-45)		-24220
8(45)		16552
	ANSYS (AVERAGE)	-2180.44
	PRESENT SOFTWARE	-2382.5
	%DIFF	8.48

Table 9: Average shear stress on the web of I-beam due to shear force

PLY	Unit : lb/in2	SXZ
1(45)	ANSYS	-8364.9
2(-45)		-12453
3(-45)		-12453
4(45)		-8364.9
	ANSYS (AVERAGE)	-10409
	PRESENT SOFTWARE	-9834.84
	%DIFF	5.84

4 CONCLUSIONS

A software that can perform stress analysis of thin-walled open sections made of composite materials has been developed using MATLAB. The results for shear force and torsion obtained from the software were found to be in good agreement with ANSYS and these tabulated results presented will be useful for validation. This software combined with the earlier software (Ali, et al., 2015) will be able to calculate ply stresses and average stresses due to axial load, bending moment and torsion, average shear stress due to shear force, axial and bending stiffness, torsional stiffness, rate of twist as well as section properties such as centroid and shear centre. Thus this software is expected to be useful as an educational tool for both students and lecturers for effective teaching-learning process.

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